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SUSY Glue-Balls, Dynamical Symmetry Breaking and Non-Holomorphic Potentials.

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Abstract

We discuss the instability of the Veneziano-Yankielowicz effective action (or its supersymmetric ground-state) with respect to higher order derivative terms. As such terms must be present in an effective action, the V-Y action alone cannot describe the dynamics of SYM consistently. We introduce an extension of this action, where all instabilities are removed by means of a much richer structure of the Kähler potential. We demonstrate that the dominant contributions to the effective potential are determined by the non-holomorphic part of the action and we prove that the non-perturbative ground-state can be equipped with stable dynamics. Making an expansion near the resulting ground-state to second order in the derivatives never leads back to the result by Veneziano and Yankielowicz. As a consequence new dynamical effects arise, which are interpreted as the formation of massive states in the boson sector (glueballs) and are accompanied by dynamical supersymmetry breaking. As this regime of the dynamics is not captured by standard semi-classical analysis (instantons etc.), our results do not contradict these calculations but investigate the physics of the system beyond these approximations.

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1 Introduction

The low-energy dynamics of $N = 1$ SYM theory are described in terms of a chiral superfield $\Phi = \varphi + \theta\psi + \theta^2 F$. Its lowest component φ is the gaugino condensate while the highest one is the scalar glue-ball operator. The effective action is given by the most general Lagrangian of this superfield obeying all symmetries. It has been shown in ref. [?] that the most general Lagrangian of a chiral superfield Ψ does not consist of polynomials of this field alone, but of polynomials in infinitely many chiral fields Ψ_n , where

$$\Psi_0 = \Psi, \quad \Psi_n = \bar{D}^2 \bar{\Psi}_{n-1}. \quad (1.1)$$

The complete (non-local) action can then be written as

$$\mathcal{L} = \int d^4x \left(\int d^4\theta K(\Psi_n, \bar{\Psi}_n) - \left(\int d^2\theta W(\Psi_n) + \text{h.c.} \right) \right), \quad (1.2)$$

where both K and W are polynomial functions, i.e. before using the constraint (1.1) they describe the standard Kähler- and superpotential, respectively. All higher derivatives arise from the constraint (1.1). In certain cases stability requirements force us to add a term of the form $\bar{\Psi}\Psi\partial_\mu\Psi\partial^\mu\bar{\Psi}$ transgressing the form (1.2). This leads to an alternative description of the model, namely

$$\mathcal{L} = \int d^4x \left(\int d^4\theta A(\Psi_0, \Psi_1, \bar{\Psi}_0, \bar{\Psi}_1) + \left(\int d^2\theta H(\Psi_0) + \text{h.c.} \right) \right), \quad (1.3)$$

where $A(\Psi_0, \Psi_1, \bar{\Psi}_0, \bar{\Psi}_1)$ and $H(\Psi_0)$ are no longer polynomial functions in the superfields but also contain space-time derivatives. This Lagrangian (1.3) describes the most general form of the effective action.

In this paper we construct a specific example of (1.2)/(1.3) obeying all symmetries and stability requirements of an effective action for $N = 1$ SYM¹. In this action the two fields Ψ_0 and Ψ_1 are given in terms of the low-energy field Φ as

$$\Psi_0 = (\Phi)^{\frac{1}{3}}, \quad \Psi_1 = \bar{D}^2 \bar{\Psi}_0. \quad (1.4)$$

The reason for the fractional power in equation (1.4) is discussed in section 4.1.

We show that the inclusion of Ψ_1 leads to important modifications compared to the restricted action of [?], which is polynomial in Ψ_0 alone:

$$\mathcal{L}_{V-Y} = \int d^4x \left(\int d^4\theta K(\Psi_0, \bar{\Psi}_0) - \left(\int d^2\theta W(\Psi_0) + \text{h.c.} \right) \right) \quad (1.5)$$

¹Some steps of this construction had been performed in ref. [?] already, but a detailed analysis of the system had not been given therein.

These modifications are a consequence new terms appearing in the non-holomorphic part of the action: The inclusion of Ψ_1 allows terms in any power of F while (1.5) is restricted to terms linear and quadratic in this field. Therefore the (physically relevant) body of the Kähler potential (in particular the Kähler metric $g_{\varphi\bar{\varphi}}$) is a function of φ and F in (1.2)/(1.3): $K = K(\varphi, F; \bar{\varphi}, \bar{F})$. This essential dependence on F allows to define a (effective) potential with massive particles and all relevant couplings even for $W \equiv 0$. Thus the spectrum of the theory is not defined by the superpotential alone. The generalized form (1.2)/(1.3) changes the physical spectrum completely compared to (1.5). Moreover our action can have any power of derivatives on all fields (including F), while (1.5) has no derivatives on F .

We will prove below that these two points must lead to an essentially different behavior of the theory than (1.5) has. The action (1.5) can never be seen as an approximation. Physical and mathematical requirements on the behavior of an effective action tell us that solely the general models (1.2) or (1.3) are acceptable as an ansatz of the latter.

The existence of higher order derivative terms and of an effective potential independent of the superpotential has important consequences for the holomorphic terms as well. Indeed, the restriction of H being a holomorphic function in the superfields Ψ_n is one of the most powerful restrictions from supersymmetry on the general form of the action. If \mathcal{L} is the fundamental Lagrangian, holomorphicity leads to the non-renormalization theorems in perturbation theory. But even in the non-perturbative region holomorphic dependence leads to severe constraints on the behavior of the theory, summarized e.g. in [?, ?]. Besides purely field-theoretical statements interesting relations to string- and M-theory are based on holomorphic structure as well: Recent results have shown how to obtain the superpotential $W(\Phi)$ of the action (1.5) from such models [?, ?, ?, ?].

Nevertheless, it is impossible to discuss the role of the superpotential without taking into account the non-holomorphic terms. This is almost trivial if the action is seen as a fundamental Lagrangian, e.g. as a Wess-Zumino model, where as an example the mass of the particles is determined by the quadratic part of the superpotential:

$$V_W = m \int d^2\theta \Phi\Phi + \text{h. c.} = m(F\varphi - \psi\psi) + \bar{m}(\bar{F}\bar{\varphi} - \bar{\psi}\bar{\psi}) \quad (1.6)$$

Obviously this expression gives a mass m to the spinor ψ , but as it stands it does not define a mass for φ . This is obtained by writing down the complete potential including terms from the non-holomorphic function $K(\Phi, \bar{\Phi}) = \bar{\Phi}\Phi$ leading to

$$V = -\bar{F}F + m(F\varphi - \psi\psi) + \bar{m}(\bar{F}\bar{\varphi} - \bar{\psi}\bar{\psi}) . \quad (1.7)$$

Completion of the square in the above equation yields the supersymmetric spectrum. The potential in F reduces to $V_F = -\bar{F}F$ and determining F by demanding the functional derivative to vanish represents a valid constraint, reducing the degrees of freedom. This procedure is allowed as F is an auxiliary field in this case, i.e. there are no space-time derivatives acting on F .

In an effective action, which must be based on (1.2) or (1.3), the above procedure is impossible as F is no longer an auxiliary field. At this point the combination of the two modifications mentioned above is important: Due to the derivatives acting on F the potential of (1.5) $V_F \propto -\bar{F}F$ becomes unstable. Thus additional non-holomorphic terms have to define a stable minimum. This is possible in (1.2)/(1.3) as the Kähler potential is a function of φ and F . This leads to dominant contributions to the potential from this part of the action, while the role of the superpotential is changed completely (see section 3 below). These effective actions have several typically non-perturbative features that cannot be found in classical or perturbative supersymmetry [?].

The role of the non-holomorphic parts of supersymmetric Lagrangians eludes a systematic study that is possible in case of the holomorphic terms. But as the dominant contributions to the effective potential of the actions (1.2)/(1.3) stem from the non-holomorphic part, we find that the ground-state of supersymmetric Yang-Mills theories is characterized by essentially non-perturbative effects, which at the present time are not available for semi-classical and/or perturbative calculations. These effects are closely related to the dynamics of the glue-ball [?], which –in contrast to chiral symmetry breaking and quark-confinement– is difficult to understand in a semi-classical picture of non-supersymmetric QCD as well. In addition dynamical supersymmetry breaking (DSB) occurs. This form of DSB cannot be understood by semi-classical concepts as instanton-induced DSB and defies perturbative and semi-classical arguments about the absence of DSB in SYM [?, ?].

We already stress at this point that these conclusions are completely independent of the role of F once the true ground-state is determined. Indeed, having found the correct ground-state we may arrive at the conclusion that the residual dynamics of F are actually suppressed and may thus be integrated out. This does *not* lead back to the interpretation of F as an auxiliary field in the sense of (1.7), but is a conclusion that applies in an expansion around the correct ground-state only, where F is *never* an auxiliary field. This seeming contradiction is the consequence of approximations versus exact mathematical statements: Integrating out a field is a question of correct approximation (or correct counting of orders of the scale Λ), but to qualify a certain field as auxiliary is an exact mathematical statement, where sub-leading effects in Λ *do matter* (cf. the discussion of section 3 and of reference [?]).

The paper is organized as follows: The derivation of the Lagrangian by Veneziano and Yankielowicz is shortly reviewed in section 2, the necessity of a generalization of this model demonstrated in section 3. In section 4 we construct the generalized model and discuss its fundamental properties and in section 5 we relate these results to independent knowledge about non-perturbative SYM theories. Finally we draw our conclusions in section 6.

2 The Veneziano-Yankielowicz Lagrangian

The effective description of $N = 1$ SYM theories goes back to the work by Veneziano and Yankielowicz [?]. The authors showed that it is possible to obtain a low energy description

of the theory by means of an effective Lagrangian in terms of the anomaly multiplet (also referred to as Lagrangian or glue-ball multiplet)

$$\Phi = \frac{1}{8C(G)} \text{Tr } W^\alpha W_\alpha = \varphi + \theta\psi + \theta^2 F \quad (2.1)$$

In the present paper we extend this Lagrangian by adding higher derivative terms and we show that this procedure changes the role of the local part of [?] in a non-trivial way. To make our statements more transparent we shortly review the arguments leading to the Lagrangian of ref. [?].

It is most natural to assume that the low-energy dynamics of confined SYM theories must be described in terms of the anomaly multiplet Φ . Indeed, its lowest component is the gaugino-condensate, the spinor represents the Goldstino if supersymmetry breaks dynamically and its highest component contains the operators $\text{Tr } F_{\mu\nu} F^{\mu\nu}$ and $\text{Tr } F_{\mu\nu} \tilde{F}^{\mu\nu}$ used to represent the anomaly of the R -current and scale invariance. Also, this multiplet has a definite interpretation in the underlying symmetry structure as it appears in the anomaly of the supercurrent²

$$\bar{D}^{\dot{\alpha}} V_{\alpha\dot{\alpha}} = \frac{2}{3C(G)} \frac{\beta(g)}{g^3} D_\alpha \Phi . \quad (2.2)$$

Thus we can expect that the multiplet Φ is defined not only in perturbation theory but also in the non-perturbative region. The effective description is obtained by writing down the most general Lagrangian in terms of Φ having the correct symmetry properties. Φ is gauge-invariant by construction and supersymmetry is realized linearly as Φ is a chiral superfield. Thus the most general Lagrangian has the form

$$\mathcal{L} = \int d^4x \left(\int d^4\theta K(\Phi, \bar{\Phi}) - \left(\int d^2\theta W(\Phi) + \text{h. c.} \right) \right) . \quad (2.3)$$

The remaining symmetries are the anomalous R -current, scale invariance and special supersymmetry transformations (S -supersymmetry). Correct anomaly cancellation is obtained by a term $\propto \Phi \log(z\Phi/\Lambda^3)$ with Λ being the scale of the theory [?, ?]. Adding up all invariant terms that are polynomial in the field Φ , we arrive at the Veneziano-Yankielowicz Lagrangian

$$\mathcal{L}_{V-Y} = \int d^4x \left(\int d^4\theta 9a(\bar{\Phi}\Phi)^{\frac{1}{3}} - \left(\int d^2\theta \frac{2\beta(g)}{9C(G)g^3} (\Phi \log \frac{z\Phi}{\Lambda^3} - \Phi) + \text{h. c.} \right) \right) . \quad (2.4)$$

Symmetries determine the Lagrangian up to three dimensionless constants a , z and the renormalized coupling constant of the fundamental theory, g . Performing the integrals over

²This supercurrent in equation (2.2) is not to be confused with the local current of the supercharge Q_α .

Grassman variables yields $(2\beta/(9C(G)g^3) = c)$

$$\begin{aligned} \int d^4\theta \, 9(\bar{\Phi}\Phi)^{\frac{1}{3}} = \\ = (\bar{\varphi}\varphi)^{-\frac{2}{3}}(\partial_\mu\bar{\varphi}\partial^\mu\varphi + \frac{i}{2}\psi\sigma^\mu\overleftrightarrow{\partial}_\mu\bar{\psi} + \bar{F}F) - \frac{i}{9}\psi\sigma^\mu\bar{\psi}(\bar{\varphi}\varphi)^{-\frac{5}{3}}(\varphi\overleftrightarrow{\partial}_\mu\bar{\varphi}) \\ + \frac{1}{9}(\bar{\varphi}\varphi)^{-\frac{5}{3}}(\psi\psi)(\bar{\psi}\bar{\psi}) + \frac{1}{3}(\varphi^{-\frac{2}{3}}\bar{\varphi}^{-\frac{5}{3}}F\bar{\psi}\bar{\psi} + \text{h. c.}) \end{aligned} \quad (2.5)$$

and

$$\int d^2\theta \, c(\Phi \log \frac{z\Phi}{\Lambda^3} - \Phi) = c(F \log \frac{z\varphi}{\Lambda^3} - \frac{1}{2\varphi}\psi\bar{\psi}) . \quad (2.6)$$

At this point Veneziano and Yankielowicz treat F as an auxiliary field and eliminate it by means of its equation of motion (in accordance with (1.7)). Indeed, varying (2.4) with respect to \bar{F}

$$F = -\frac{1}{3}\frac{\psi\bar{\psi}}{\varphi} + \frac{(\bar{\varphi}\varphi)^{\frac{2}{3}}}{a}c \log \frac{\bar{z}\bar{\varphi}}{\Lambda^3} \quad (2.7)$$

and inserting this back into the Lagrangian an expression completely independent of F –and thus independent of $\text{Tr } F_{\mu\nu}F^{\mu\nu}$ and $\text{Tr } F_{\mu\nu}\tilde{F}^{\mu\nu}$ – is obtained:

$$\begin{aligned} \mathcal{L}_{V.Y} = \int d^4x \, a(\bar{\varphi}\varphi)^{-\frac{2}{3}}(\partial_\mu\bar{\varphi}\partial^\mu\varphi + \frac{i}{2}\psi\sigma^\mu\overleftrightarrow{\partial}_\mu\bar{\psi}) - \frac{ia}{9}\psi\sigma^\mu\bar{\psi}(\bar{\varphi}\varphi)^{-\frac{5}{3}}(\varphi\overleftrightarrow{\partial}_\mu\bar{\varphi}) \\ - \frac{(\bar{\varphi}\varphi)^{\frac{2}{3}}}{a}c^2 \log \frac{z\varphi}{\Lambda^3} \log \frac{\bar{z}\bar{\varphi}}{\Lambda^3} + c\left((\psi\psi)\left(\frac{1}{3\varphi} \log \frac{\bar{z}\bar{\varphi}}{\Lambda^3} + \frac{1}{2\varphi}\right) + \text{h. c.}\right) \end{aligned} \quad (2.8)$$

After having eliminated the auxiliary field the chirally broken minimum with $\varphi_0 = \Lambda^3/z$ is found. Notice again that this minimum is not obtained from the superpotential (2.6) alone. The non-holomorphic terms in F from (2.5) play a crucial role to arrive at this conclusion.

The complex number z determines the phase of φ_0 . In this work we consider the action as a quantum effective action obtained from a source-extension (discussed in section 4 below). Vacuum alignment then dynamically determines the phase of φ_0 : $\varphi_0 < 0$ and thus z must be negative. This follows from PCAC analysis [?] or QCD effective actions [?], in more general situations a complete study of thermodynamical limits [?, ?, ?] may be important. Thus we set z in the following to $z = -1$. Making an expansion around this minimum and absorbing the constant in front of the kinetic term by a field redefinition

$$\varphi' = \frac{\sqrt{a}}{\Lambda^2}\varphi , \quad \psi' = \frac{\sqrt{a}}{\Lambda^2}\psi , \quad (2.9)$$

one finds the supersymmetric spectrum with $m = (c/a)\Lambda$ as masses for both, φ and ψ .

3 From Local to Non-Local Actions

It is of utmost importance to observe that all conclusions from the work by Veneziano and Yankielowicz –unbroken supersymmetry, confining vacuum state with spectrum, positivity property of the potential– can be drawn *if and only if* the field F is auxiliary. We prove in this section that this statement must be an exact mathematical requirement on the full theory (including all terms beyond the approximation (2.4) or (2.8)). Thus in any enveloping theory, of which (2.8) shall be an approximation, F remains an auxiliary field *exactly*.

We will prove below that this restriction is untenable, as the Veneziano-Yankielowicz Lagrangian must be extended by additional terms promoting F to an independent physical degree of freedom. Indeed, we cannot expect that this action represents the complete dynamics at low energies: it has derivative terms up to $\mathcal{O}(p^2)$ only, which is not acceptable for an interacting theory. Generalizing (2.4) we write

$$\mathcal{L}_{SYM}(\Phi, \bar{\Phi}, \dots) = \mathcal{L}_{V-Y}(\Phi, \bar{\Phi}) + \Delta\mathcal{L}(\Phi, \bar{\Phi}, \dots) , \quad (3.1)$$

where the dots indicate that the full effective action could depend on additional fields. The subsequent arguments of this section do not depend on the approximation techniques as exemplified by the Wilsonian action, which is also of the form (3.1).

Whether \mathcal{L}_{V-Y} is a good approximation of the whole theory or not depends on the details of $\Delta\mathcal{L}$. Usually it is assumed that such an expansion up to second order in the derivatives leads to a good approximation if all neglected terms are suppressed by inverse powers of Λ . But stability/instability is not just a question of the order of contributions from $\Delta\mathcal{L}$: If \mathcal{L}_{V-Y} shall be an approximation of the whole theory, we thus have to ensure that the fundamental behavior of the F field (namely being auxiliary) is not changed by $\Delta\mathcal{L}$.

The field F has mass dimension four and suppressing the dependence on the additional fields present (or setting them to their ground-state values $\varphi = \varphi_0$ and $\psi \equiv 0$) the Lagrangian by Veneziano and Yankielowicz remains of the form (1.7)

$$\mathcal{L}_F = \frac{c_1}{\Lambda^4} \bar{F} F - dF - \bar{d}\bar{F} . \quad (3.2)$$

Considering the stability of the system the linear terms are irrelevant. The potential in F is not bounded from below but instead has an absolute maximum, the typical behavior of an auxiliary field. The supersymmetric spectrum obtained by Veneziano and Yankielowicz is found by sitting on the top of the auxiliary-field potential, which only after elimination of F becomes the new minimum of the physical potential.

We now add additional contributions from $\Delta\mathcal{L}$ to the action. They should start at order $\mathcal{O}(\Lambda^{-6})$ and can include derivative terms as well as modifications of the potential. Allowing additional derivative terms from $\Delta\mathcal{L}$, e.g. a new term

$$\mathcal{L}_F = \frac{|c_2|}{\Lambda^6} \partial_\mu F \partial^\mu \bar{F} + \frac{c_1}{\Lambda^4} \bar{F} F - dF - \bar{d}\bar{F} \quad (3.3)$$

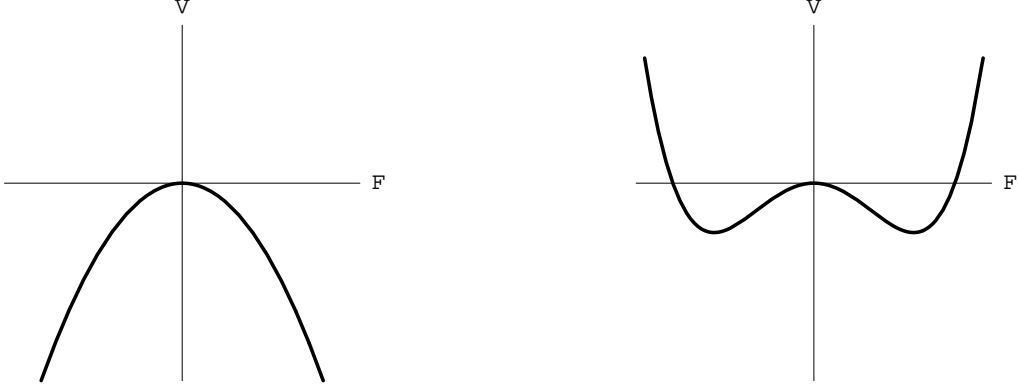


Figure 1: Additional terms to the potential of F can turn the unstable potential from the Veneziano-Yankielowicz Lagrangian (left hand side) into an acceptable potential for dynamical F (right hand side). If F is auxiliary, the ground state is represented by the maximum of the Veneziano-Yankielowicz potential, if F is dynamical the latter is represented by the minimum of the new potential.

could appear, disqualifying the interpretation of F as auxiliary field. Of course the kinetic term in F is formally suppressed compared to the kinetic terms of φ and ψ , which are of order $\mathcal{O}(\Lambda^{-4})$. But at this point it is important to notice that the stability of a system and the characterization of certain fields as auxiliary ones cannot be restricted to some approximation but are *exact mathematical statements*. If c_2 –or any higher order derivative in F – is non-zero, F is no longer an auxiliary field. Of course, this behavior has nothing to do with supersymmetry, but is a simple statement about classical mechanics: We put a ball on the top of a hill. When hitting it with an arbitrarily small but finite momentum (i.e. some dynamics that can be suppressed by any finite power of the typical scale (mass of the ball) of the system) this inevitably leads to a dynamical revelation, i.e. the ball will roll down the hill. A different conclusion is allowed if and only if the ball is an “auxiliary ball”. This means that the dynamics have to vanish exactly *in the full theory* and we can’t hit the ball at all, not even with an infinitesimal momentum.

If c_2 (or any other term from $\Delta\mathcal{L}$ including derivatives on F) is non-zero, the only way out is to provide for a finite bottom of the hill in (3.3) (cf. figure 1). If we assume that the action (3.3) is complete up to order $\mathcal{O}(\Lambda^{-4})$ the simplest solution is

$$\mathcal{L}_F = \frac{|c_2|}{\Lambda^6} \partial_\mu F \partial^\mu \bar{F} + \frac{c_1}{\Lambda^4} \bar{F} F - \frac{|c_3|}{\Lambda^{12}} (\bar{F} F)^2 - dF - \bar{d}\bar{F} . \quad (3.4)$$

Now our ball will still roll down the hill, but at least this will stop when it reaches the new

minimum ($d = 0$ in the following)

$$F_0 = \Lambda^4 \sqrt{\frac{|c_1|}{2|c_3|}}, \quad (3.5)$$

which is simply the minimum of a Mexican-hat potential. In this new minimum we may still arrive at the conclusion that the dynamics of F are suppressed and that we may integrate this field out in a low-energy approximation (notice that an application of this idea to supersymmetry won't lead to a Goldstone mode from F). But this is now an approximation allowed in the vicinity of the ground-state (3.5), only. The correct ground-state is given by (3.5) and the original interpretation of F being an auxiliary field (cf. (3.2)) is *never* an approximation to the true behavior of the theory.

Equation (3.4) describes an effective Mexican-hat of some field with mass dimension four. Abstracting from supersymmetry we once again can make absolutely transparent what happens: The full dynamics of the system tell us that the minimum of the potential is (up to a phase) at F_0 of equation (3.5). Sitting in this minimum we obtain the correct physical spectrum of the theory and may integrate out some fields if they are really suppressed. Standing on the top of the hill however, power counting and suppression arguments are absolutely fallacious. Else we would arrive at the conclusion that both c_2 and c_3 are irrelevant effects and the theory shrinks to the action (3.3). Obviously this does not at all describe the correct physics of a Mexican-hat like potential as the system is expanded around an unstable point.

The question whether (2.8) is a suitable approximation of the low-energy dynamics of SYM or not thus depends on the details of the full theory. Of main importance is the question whether $\Delta\mathcal{L}$ really contains derivative terms on the field F or not. Such terms must be present due to the following two points:

1. Supersymmetry is realized linearly. This is one of the basic assumptions made when writing down the Lagrangian (2.4). But again, this is a statement about the whole effective action. Thus not solely (2.4), but also $\Delta\mathcal{L}$ must be an integral over superspace.

If the low energy dynamics are described by Φ alone (e.g. in a quantum effective action), higher derivatives on φ and ψ inevitably lead to derivative terms in F . When introducing additional fields, we could try to arrange them in such a way that all derivative terms in the auxiliary fields cancel (such a model has been derived in ref. [?] as an ansatz for the effective action of SUSY QCD). Using this idea in pure SYM would imply that the new fields are not suppressed compared to Φ , in contradiction to our assumption about the low-energy dynamics of SYM. Thus it is impossible to keep F auxiliary in a non-local Lagrangian, if the low-energy dynamics shall depend on the three fields φ , ψ and F , only.

2. The effective action must have higher order derivative terms that are not present in the Veneziano-Yankielowicz Lagrangian. A local effective action is not acceptable for an interacting theory.

Besides this general objection a careful analysis within the supersymmetric framework shows that the locality of supersymmetric non-linear sigma models is not just a harmless peculiarity when using them in effective descriptions. Indeed, the low energy spectrum should include the glue-ball. The only candidate for a glue-ball operator is $F_{\mu\nu}F^{\mu\nu}$, the real part of F . If F remains auxiliary we have to assume that the glue-ball of $N = 1$ SYM does not belong to the relevant low-energy degrees of freedom. Even if this assumption is correct the relation to pure YM theory remains mysterious. If the low-energy dynamics shall be described in terms of renormalizable operators (or their classical fields, resp.) pure YM theory does generate non-trivial dynamics for $F_{\mu\nu}F^{\mu\nu}$ and we are left with the unsatisfactory situation that either the two classical fields $\langle\Omega(J)|F_{\mu\nu}F^{\mu\nu}|\Omega(J)\rangle_{\text{SYM}}$ and $\langle\Omega(J)|F_{\mu\nu}F^{\mu\nu}|\Omega(J)\rangle_{\text{YM}}$ are completely independent objects or that there exists a phase transition in the decoupling of the gluino by giving it a heavy mass (its origin and consequences have been discussed in [?]). It has been tried to escape this restriction by the choice of a more complicated geometry than realized in (2.4) [?]. From our point of view the result of [?] cannot lead to a resolution of the problem [?]. Moreover, trying to add higher order derivatives to the action of [?] leads to the same complications as discussed in this work for the Veneziano-Yankielowicz Lagrangian and their resolution automatically leads to the inclusion of the glue-ball dynamics (see next section). Thus the construction of [?] becomes redundant in the context of non-local actions.

As a consequence of (1) and (2) F has to be seen as an independent degree of freedom. At least in the framework of effective actions it is possible to formulate consistent supersymmetric actions of this type [?], where some important features of supersymmetric theories with “normal” auxiliary fields are compromised. This especially includes (see ref. [?] for a detailed discussion):

1. The full potential is not positive semi-definite: Obviously the positivity of the potential holds after elimination of the auxiliary fields, only. The statement that supersymmetric potentials are always positive semi-definite is actually a statement about the maximum of the auxiliary field potential: The latter is zero (unbroken supersymmetry) or positive (broken supersymmetry). As this maximum does no longer represent the ground state in a theory with dynamical “auxiliary” fields, the physical potential can have a negative minimum. This has important consequences in the discussion of supersymmetry breaking: In classical and perturbative supersymmetry a supersymmetric state (i.e. a state with vanishing potential) is automatically the ground-state and implies unbroken supersymmetry. In the non-perturbative region this conclusion does not hold, as the “auxiliary” fields become dynamical.

2. While the potential is not necessarily positive, the restriction on the vacuum energy still holds: $\langle \Omega | T_{\mu\nu} | \Omega \rangle = g_{\mu\nu} E_0 \geq 0$ due to supersymmetry current-algebra relations [?]. $E_0 > 0$ implies supersymmetry breaking, while $E_0 = 0$ means unbroken supersymmetry. In a strictly perturbative and/or semiclassical logic a strict relation between the minimum of the effective potential and the order parameter E_0 exists: $(V_{\text{eff}})_0 \equiv E_0$ due to the non-renormalization theorem. In a fully non-perturbative framework an extension of this relation to $(V_{\text{eff}})_0 \neq 0$, $E_0 \neq 0$ but $(V_{\text{eff}})_0 \neq E_0$ is possible. A consistent relation of this form can be achieved only by a non-perturbative extension of the trace anomaly (2.2)³. But in any case the order parameter of supersymmetry breaking is still E_0 and in the effective theory the latter is represented by the value of F in the ground-state.
3. Supersymmetry breaks dynamically. Interestingly enough this follows directly from (2.4) and (2.8): The unstable maximum is at the supersymmetry preserving point (vanishing potential and $F = 0$). As the action (2.4) is complete up to order $\mathcal{O}(\Lambda^{-4})$, the unstable maximum cannot turn directly into a minimum (stability of $\mathcal{O}(p^2)$ forbids to choose $a < 0$ in (2.4)). Instead, the new minimum must be at a different value of F : $F_0 \neq 0$. But F is the order parameter of supersymmetry breaking and we arrive at the desired result. The argument of [?] that supersymmetry is unbroken in $N = 1$ SYM thus turns into the opposite when taking into account all (including non-local) terms.

4 Non-Local Effective Action of SYM

As outlined above the extension of the Veneziano-Yankielowicz Lagrangian to a non-local model with dynamical “auxiliary” field changes its fundamental properties. Thus the explicit form of such an extension must be studied, even if most of the terms will be unimportant in any application. To this end, a more rigorous definition of the object discussed here should be given first.

4.1 Definition and Symmetries of the Action

In this work we consider a first step in the discussion of non-perturbative SYM theories. It consists in finding the correct ground-state. The latter is defined as a thermodynamical limiting process of the quantum effective action leading to the minimum of the effective potential (see sect. 2 of ref. [?] and references therein). Thus a quantum effective action must be defined, where the components of Φ represent the classical fields. A quantum effective action with this field content is obtained by extending the complex coupling constant τ to a

³In this paper we do not provide such an extension.

chiral superfield of sources [?, ?]

$$J(x) = \tau(x) + \theta\eta(x) - 2\theta^2 m(x) . \quad (4.1)$$

The effective action is then defined as the Legendre transform with respect to these sources

$$\Gamma[\tilde{J}, \tilde{\bar{J}}] = \int d^4x \left(J(x) \frac{\delta W[J]}{\delta J(x)} + \text{h. c.} \right) - W[J, \bar{J}] . \quad (4.2)$$

\tilde{J} are the classical fields φ_{cl} , ψ_{cl} and F_{cl} . As they shall transform linearly under supersymmetry they can be combined to the chiral multiplet Φ of equation (2.1) (we suppress the index “cl” in the following). (4.2) together with (4.1) defines the unique effective action in terms of gauge-invariant and supersymmetry covariant classical fields. In the thermodynamical limiting process

$$\frac{\delta}{\delta \tilde{J}} \Gamma[\tilde{J}, \tilde{\bar{J}}] = J(x) \rightarrow 0 \quad \frac{\delta}{\delta J(x)} W[J, \bar{J}] = \tilde{J}(x) \rightarrow \tilde{J}^* \quad (4.3)$$

this effective action (or its effective potential resp.) lead per definitionem to the true ground-state of the theory. Equation (4.3) has to be understood as a limiting process starting from a suitable UV and IR regularization and performing the full non-perturbative renormalization. During this process *all* regularizations are getting removed and the sources are finally relaxed to $J(x) \rightarrow 0$. The dual parameters (classical fields) adopt their correct vacuum expectation values in the limit.

Of course, we cannot perform these calculations explicitly. Thus certain assumptions enter in writing down equation (4.2) as discussed in [?] and references therein.

Our task is therefore to extend the action (2.4) to a non-local model, taking into account the important new features appearing therein according to the discussion of the last section. This non-local action is then seen as an ansatz for the quantum effective action (4.2).

To extend the model (2.4) to a non-local Lagrangian we use as starting-point the result of [?]. Therein the authors showed that the action (2.3) is –within the context of effective field theories– a restricted form of a more general model, which is mandatory and of the form

$$\mathcal{L} = \int d^4x \left(\int d^4\theta K(\Psi_n, \bar{\Psi}_n) - \left(\int d^2\theta W(\Psi_n) + \text{h. c.} \right) \right) . \quad (4.4)$$

The index n runs from zero to infinity, the defining field is $\Psi_0 \equiv \Psi$ and the higher Ψ_n are related to Ψ_0 by

$$\Psi_n = \bar{D}^2 \bar{\Psi}_{n-1} , \quad \Psi_{2n} = (-1)^n \square^n \Psi_0 , \quad \Psi_{2n+1} = (-1)^n \square^n \Psi_1 . \quad (4.5)$$

Of course, the Lagrangian in (4.4) could be formulated in terms of Ψ_0 and Ψ_1 alone when dropping the assumption that the Kähler- and superpotential are polynomials in Ψ , but are allowed to include explicit space-time derivative terms. The corresponding action (1.3)

$$\mathcal{L} = \int d^4x \left(\int d^4\theta A(\Psi_0, \Psi_1, \bar{\Psi}_0, \bar{\Psi}_1) - \left(\int d^2\theta H(\Psi_0) + \text{h. c.} \right) \right) \quad (4.6)$$

is actually more general than (4.4). However, the effective potential from (4.4) and (4.6) are the same, which is the quantity we are mostly interested in. The advantage of (4.4) is its simple mathematical form, but in certain cases stability requirements force us to add one term of order $\mathcal{O}(p^2)$ that transgresses this form (see section 4.3 below).

The defining superfield of our model is the classical field Φ obtained from Legendre transformation. Nevertheless it is difficult to implement all symmetries when choosing $\Psi = \Phi$. Standard superconformal transformations of a chiral superfield imply that the mass dimension and the R weight of such a field obey the relation $d = (3/2)r$. Of course, Φ is a correct chiral superfield in this sense, it has mass dimension 3 and chiral weight 2. But $\bar{D}^2\bar{\Phi}$ has mass dimension 4 and chiral weight 0. The field still has standard transformations under R symmetry and scale transformations, but not under S-supersymmetry. Thus an action in terms of Φ and $\bar{D}^2\bar{\Phi}$ invariant under R and scale transformations would not be invariant under S-supersymmetry.

We can escape these difficulties by observing that a chiral field C of the form $C = \bar{D}^2\bar{B}$ transforms under S-supersymmetry with dimension 2, regardless of the actual mass dimension of C . If the true dimension of C is not equal to 2, there appear additional anomalous transformation terms. Thus we recover standard transformations under the full superconformal group by choosing

$$\Psi_0 = (\Phi)^{\frac{1}{3}}. \quad (4.7)$$

A detailed analysis of the superconformal Ward identities, leading to the defining superfields Ψ_0 and Ψ_1 as given in (4.7), has been performed in [?].

Of course, the third root of equation (4.7) has a threefold multivaluedness. The consequential ambiguities in the invariant part of the Lagrangian are the same as in (2.4). Due to the relation (4.7) the WZ-term of the actions (4.4) and (4.6) is restricted to the result found by Veneziano and Yankielowicz, which does not show an ambiguity.

Kähler and superpotential in (4.4) are simply polynomials in the fields Ψ_n and thus we may write the invariant part of this action as

$$K(\Psi_n, \bar{\Psi}_n) = \sum_{\xi\bar{\xi}} \alpha_{\xi\bar{\xi}} (\Psi_0)^{\xi_0} \dots (\Psi_\infty)^{\xi_\infty} (\bar{\Psi}_0)^{\bar{\xi}_0} \dots (\bar{\Psi}_\infty)^{\bar{\xi}_\infty}, \quad (4.8)$$

$$W(\Psi_n) = \sum_{\chi} \beta_{\chi} (\Psi_0)^{\chi_0} \dots (\Psi_\infty)^{\chi_\infty}. \quad (4.9)$$

The theory depends on infinitely many dimensionless coupling constants $\alpha_{\xi\bar{\xi}}$ and β_{χ} . The vectors ξ , $\bar{\xi}$ and χ are restricted to combinations with correct mass dimension and R invariance:

$$\sum_i (i+1)(\xi_i + \bar{\xi}_i) = 2 \quad \sum_i ((\xi_{2i} - \bar{\xi}_{2i}) + 2(\xi_{2i+1} - \bar{\xi}_{2i+1})) = 0 \quad (4.10)$$

$$\sum_i (i+1)\chi_i = 3 \quad \sum_i (\chi_{2i} + 2\chi_{2i+1}) = 3 \quad (4.11)$$

In principle each term of the action can now be calculated from the θ -expansion of Ψ_0 and Ψ_1 by using the recursive formulas (4.5). We denote the components of Ψ_0 by a hat. In terms of the defining fields φ , ψ and F these fields are given by

$$\begin{aligned}\Psi_0 &= \hat{\varphi} + \theta\hat{\psi} + \theta^2\hat{F} \\ &= \varphi^{\frac{1}{3}} + \frac{1}{3}\varphi^{-\frac{2}{3}}\theta\psi + \frac{1}{3}\theta^2\left(\varphi^{-\frac{2}{3}}F + \frac{1}{3}\varphi^{-\frac{5}{3}}(\psi\psi)\right)\end{aligned}\tag{4.12}$$

$$\begin{aligned}\Psi_1 &= \hat{\bar{F}} - i\theta\sigma^\mu\partial_\mu\hat{\bar{\psi}} - \theta^2\Box\hat{\bar{\varphi}} \\ &= \frac{1}{3}(\bar{\varphi}^{-\frac{2}{3}}\bar{F} + \frac{1}{3}\bar{\varphi}^{-\frac{5}{3}}(\bar{\psi}\bar{\psi})) - \frac{i}{3}\theta\sigma^\mu\partial_\mu(\bar{\varphi}^{-\frac{2}{3}}\bar{\psi}) - \theta^2\Box\bar{\varphi}^{\frac{1}{3}}\end{aligned}\tag{4.13}$$

A similar closed form for the action (4.6) cannot be given, but the explicit derivation of some parts thereof is straightforward on the same lines. (4.8) and (4.9) together with (4.10) and (4.11) define the invariant part of the effective action, only. Adding a term generating the correct anomalies terminates the construction of the effective action.

4.2 The Effective Potential

In this section we concentrate on the effective potential and derive different minimum and consistency conditions thereof. In section (4.3) below we prove that stable dynamics around any of its consistent minima can be defined.

As all Ψ_n with $n > 1$ vanish for static configurations the relevant ξ , $\bar{\xi}$ and χ of eqs. (4.8) and (4.9) are simply two-vectors. Implementing the constraints (4.10)/(4.11), the invariant contributions from Kähler and superpotential to the effective potential read:

$$K(\Psi_l, \bar{\Psi}_l)|_{p=0} = \sum_{m,n} \alpha_{mn} \Psi_0^{1-2m} \Psi_1^m \bar{\Psi}_0^{1-2n} \bar{\Psi}_1^n \tag{4.14}$$

$$W(\Psi_l)|_{p=0} = \sum_k \beta_k \Psi_0^{3-2k} \Psi_1^k \tag{4.15}$$

Integrating out superspace yields the effective potential in terms of the components of Ψ [?]:

$$\begin{aligned}V_{\text{eff}} &= -g_{\hat{\varphi}\hat{\varphi}}\hat{F}\hat{\bar{F}} + \frac{1}{2}g_{\hat{\varphi}\hat{\varphi},\hat{\varphi}}\hat{F}(\hat{\psi}\hat{\psi}) + \frac{1}{2}g_{\hat{\varphi}\hat{\varphi},\hat{\varphi}}\hat{\bar{F}}(\hat{\psi}\hat{\psi}) - \frac{1}{4}g_{\hat{\varphi}\hat{\varphi},\hat{\varphi}\hat{\varphi}}(\hat{\psi}\hat{\psi})(\hat{\psi}\hat{\psi}) \\ &\quad + \hat{F}W_{,\hat{\varphi}} + \hat{\bar{F}}\bar{W}_{,\hat{\varphi}} - \frac{1}{2}(\hat{\psi}\hat{\psi})W_{,\hat{\varphi}\hat{\varphi}} - \frac{1}{2}(\hat{\psi}\hat{\psi})\bar{W}_{,\hat{\varphi}\hat{\varphi}}\end{aligned}\tag{4.16}$$

We use the notation of [?], where $\hat{\varphi}$ refers to derivatives with respect to Ψ_0 and $\hat{\bar{F}}$ to derivatives of Ψ_1 . With (4.14) and (4.15) the explicit expression for (4.16) in terms of the

defining fields becomes⁴:

$$\begin{aligned}
V_{\text{eff}} = & - \sum_{m,n} \tilde{\alpha}_{mn} \cdot (\bar{\varphi}\varphi)^{-\frac{2}{3}(m+n+1)} \bar{F}^{m+1} F^{n+1} \cdot \\
& \cdot \left(1 + \frac{m+n+1}{3} ((\bar{F}\bar{\varphi})^{-1}(\bar{\psi}\bar{\psi}) + (F\varphi)^{-1}(\psi\psi)) \right. \\
& \quad \left. + \frac{(m+n+1)^2}{9} (\bar{F}F)^{-1}(\bar{\varphi}\varphi)^{-1}(\psi\psi)(\bar{\psi}\bar{\psi}) \right) \\
& + \sum_k \tilde{\beta}_k \cdot (\bar{\varphi}\varphi)^{-\frac{2}{3}k} F \bar{F}^k \cdot \\
& \cdot \left(1 + \frac{k}{3} ((\bar{F}\bar{\varphi})^{-1}(\bar{\psi}\bar{\psi}) + (F\varphi)^{-1}(\psi\psi)) + \frac{k^2}{9} (\bar{F}F)^{-1}(\bar{\varphi}\varphi)^{-1}(\psi\psi)(\bar{\psi}\bar{\psi}) \right) \\
& + \sum_l \tilde{\beta}_l \cdot (\bar{\varphi}\varphi)^{-\frac{2}{3}l} F^l \bar{F} \cdot \\
& \cdot \left(1 + \frac{l}{3} ((\bar{F}\bar{\varphi})^{-1}(\bar{\psi}\bar{\psi}) + (F\varphi)^{-1}(\psi\psi)) + \frac{l^2}{9} (\bar{F}F)^{-1}(\bar{\varphi}\varphi)^{-1}(\psi\psi)(\bar{\psi}\bar{\psi}) \right)
\end{aligned} \tag{4.17}$$

Hermiticity requires $(\tilde{\alpha}_{mn})^\dagger = \tilde{\alpha}_{nm}$ and $(\tilde{\beta}_l)^\dagger = \tilde{\beta}_l$. Also we have absorbed some numerical factors in the dimensionless coupling constants. The relations to the quantities appearing in eqs. (4.14) and (4.15) are:

$$\tilde{\alpha}_{mn} = \frac{(2m-1)(2n-1)}{3^{m+n+2}} \alpha_{mn} \qquad \tilde{\beta}_k = \frac{1}{3^{k+1}} \beta_k \tag{4.18}$$

Of course the effective potential (4.17) is subject to additional constraints to be discussed below. Nevertheless, we want to make some comments on this most general form:

- Already eq. (4.17) shows that the complete Veneziano-Yankielowicz Lagrangian (2.4) is part of the generalized non-local version as well. Indeed, the Kähler potential and the invariant part of the superpotential are given by the terms $\propto \alpha_{00}$ and $\propto \beta_0$.
- Most terms from the superpotential (4.15) are redundant as they appear in the Kähler potential already. This is not surprising as a superpotential $\propto (\Psi_1)^n$ can be transformed into a full superspace integral $\propto \Psi_0(\Psi_1)^{n-1}$, which is now a contribution to the Kähler potential. Therefore we will set the invariant part of the superpotential to zero in the following, except for the linear term $\propto \beta_0$. To avoid linear contributions from the WZ-term $\tilde{\beta}_0 = -c = -2\beta/(9C(G)g^3)$ must be chosen.

⁴The ambiguities showing up in the following equation deserve a special discussion –here, in QCD as well as in the Veneziano-Yankielowicz Lagrangian– deferred for the time being.

To abbreviate the remaining invariant effective potential we define a new metric in the fundamental fields φ and F :

$$\tilde{g}_{\varphi\bar{\varphi}} = \sum_{m,n} \tilde{\alpha}_{mn} (\bar{\varphi}\varphi)^{-\frac{2}{3}(m+n+1)} F^n \bar{F}^m = \frac{1}{9} (\bar{\varphi}\varphi)^{-\frac{2}{3}} g_{\hat{\varphi}\hat{\varphi}} \quad (4.19)$$

Then the invariant effective potential again takes the simple form as given in (4.16). Finally, adding the WZ-term the complete effective potential becomes

$$\begin{aligned} V_{\text{eff}} = & -\tilde{g}_{\varphi\bar{\varphi}} F \bar{F} + \frac{1}{2} \tilde{g}_{\varphi\bar{\varphi},\bar{\varphi}} F (\bar{\psi}\bar{\psi}) + \frac{1}{2} \tilde{g}_{\varphi\bar{\varphi},\varphi} \bar{F} (\psi\psi) - \frac{1}{4} \tilde{g}_{\varphi\bar{\varphi},\varphi\bar{\varphi}} (\psi\psi)(\bar{\psi}\bar{\psi}) \\ & + c \left(F \log \frac{z\varphi}{\Lambda^3} + \bar{F} \log \frac{\bar{z}\bar{\varphi}}{\Lambda^3} - \frac{1}{2\varphi} (\psi\psi) - \frac{1}{2\bar{\varphi}} (\bar{\psi}\bar{\psi}) \right) . \end{aligned} \quad (4.20)$$

Choosing $\tilde{g}_{\varphi\bar{\varphi}} = (\bar{\varphi}\varphi)^{-\frac{2}{3}}$, (4.20) reduces exactly to the potential of (2.4). But in contrast to (2.4) $\tilde{g}_{\varphi\bar{\varphi}}$ is now a function of φ and F , where any –positive or negative– power in F may appear. Therefore it is possible to obtain extrema of (4.20) that are minima in both, φ and F .

Starting from equation (4.20) these extrema and the corresponding stability conditions are equivalent to those derived in [?], when replacing the original metric $g_{\hat{\varphi}\hat{\varphi}}$ by the new one $\tilde{g}_{\varphi\bar{\varphi}}$. The minima in the field F are found for

$$\begin{aligned} \tilde{g}_{\varphi\bar{\varphi}} \bar{F} + \tilde{g}_{\varphi\bar{\varphi},F} F \bar{F} \Big|_{F=F_0} &= c \log \frac{z\varphi}{\Lambda^3} , \\ \tilde{g}_{\varphi\bar{\varphi}} + \tilde{g}_{\varphi\bar{\varphi},F\bar{F}} F \bar{F} + (\tilde{g}_{\varphi\bar{\varphi},F} F + \text{h.c.}) \Big|_{F=F_0} &< 0 . \end{aligned} \quad (4.21)$$

The complex number z must again be chosen according to the discussion in section 2. If $F_0 \neq 0$ we may replace the second constraint by

$$\tilde{g}_{\varphi\bar{\varphi}} - \tilde{g}_{\varphi\bar{\varphi},F\bar{F}} F \bar{F} - c \left(F^{-1} \log \frac{z\varphi}{\Lambda^3} + \text{h.c.} \right) \Big|_{F=F_0} > 0 . \quad (4.22)$$

The analogue conditions for φ are

$$\bar{F} \varphi \tilde{g}_{\varphi\bar{\varphi},\varphi} \Big|_{\varphi=\varphi_0} = c , \quad F \bar{F} \tilde{g}_{\varphi\bar{\varphi},\varphi\bar{\varphi}} \Big|_{\varphi=\varphi_0} < 0 . \quad (4.23)$$

The constraint $\tilde{g}_{\varphi\bar{\varphi},\varphi\bar{\varphi}} < 0$ follows from the stability of the four-Fermi interactions as well. The mass of ψ –the Goldstino– vanishes due to the first constraint in (4.23). This is an alternative proof of dynamical supersymmetry breaking. A consistent ground-state must obey the constraints (4.21) and (4.23). This is possible if and only if there exists a massless spinor ψ . But this must be a Goldstone mode if the theory is not free (even with a “supersymmetric” spectrum with both, ψ and φ massless [?]). There is an unique consequence:

supersymmetry breaks dynamically. Note that the above steps do not assume a certain value for F_0 . $F_0 \neq 0$ can be seen as a consequence of the condition (4.22) and the dynamics of ψ .

Equations (4.21)-(4.23) illustrate the essential change of the role of holomorphic and non-holomorphic parts of the effective potential. Indeed, a consistent spectrum (massive spectra for φ and F) is possible even for $c = 0$ (i.e. vanishing superpotential). In contrast to the Veneziano-Yanlielowicz model (sect. 2) the superpotential reduces to the generator of the anomalies but *never* defines stable couplings in all fields. The stabilization of the potential for large values of the fields essentially includes non-holomorphic couplings. As we cannot eliminate the “auxiliary” fields –which can now be seen as a consequence of the first equation in (4.21)– these couplings must stem from the Kähler potential alone.

Finally we should implement two constraints from the underlying dynamics. First, current algebra relations of supersymmetry tell us that $F_0 \geq 0$ and second $\varphi_0 < 0$ due to vacuum alignment. Obviously it is impossible to reduce these two conditions to simple conditions on the coupling constants $\tilde{\alpha}_{mn}$. An appealing (though not necessary) condition is to set all off-diagonal terms of α_{mn} to zero:

$$\alpha_{mn} = 0 \quad (m \neq n) , \quad z = -1 . \quad (4.24)$$

The potential then reduces to

$$\begin{aligned} V_{\text{eff}} = & - \sum_m \tilde{\alpha}_{mm} \cdot (\bar{\varphi}\varphi)^{-\frac{2}{3}(2m+1)} (\bar{F}F)^{m+1} \cdot \\ & \cdot \left(1 + \frac{2m+1}{3} ((\bar{F}\bar{\varphi})^{-1}(\bar{\psi}\bar{\psi}) + (F\varphi)^{-1}(\psi\psi)) + \frac{(2m+1)^2}{9} (\bar{F}F)^{-1}(\bar{\varphi}\varphi)^{-1}(\psi\psi)(\bar{\psi}\bar{\psi}) \right) \\ & + c \left(F \log\left(-\frac{\varphi}{\Lambda^3}\right) + \bar{F} \log\left(-\frac{\bar{\varphi}}{\Lambda^3}\right) - \frac{1}{2\varphi}(\psi\psi) - \frac{1}{2\bar{\varphi}}(\bar{\psi}\bar{\psi}) \right) \quad (4.25) \end{aligned}$$

and the remaining coupling constants still have to be arranged in such a way that a minimum with $F_0 > 0$ results.

The principle steps of this section leading to the effective potential (4.20) had been performed in [?] already. However, the author did not derive the minimum conditions (4.21)-(4.23), but expected a spectrum with unbroken supersymmetry ($F_0 = 0$). Excluding inverse powers of F he concluded that the model must be unstable and he did not find the correct stable ground-state. This agrees with our analysis of [?] and of the present work. Indeed, the description breaks down as $F \rightarrow 0$.

4.3 Consistent Dynamics

In the last section we discussed the properties of the non-perturbative effective potential, where F becomes a dynamical variable. However, we have not yet shown that these dynamics can be introduced consistently when starting from an effective potential of the form (4.20).

In this section we now prove that this is possible for any ground-state consistent with the constraints derived in the last section. We cannot fix all coefficients in a momentum-expansion around some assumed ground-state as well as we are not able to determine the correct choice of the coupling constants α_{mn} . However, this is not of main importance: While we do know that some choice of the α_{mn} leads to the correct ground-state, the effective action may not include the whole physical dynamics around this state. But we do know that there are some dynamics and that they must be stable (see ref. [?] for a detailed discussion).

In reference [?] it has been shown that acceptable dynamics of this type are possible if the defining fields are the fields appearing in the action (4.4) or (4.6). In other words, the existence of stable dynamics is known on the level of the fields $\hat{\varphi}$, $\hat{\psi}$ and \hat{F} . The strategy is as follows: First the dynamics of the action from (4.14) are calculated. That means, we determine the effects of non-static field configurations in the Kähler potential (4.14). These dynamics are expressed in terms of geometric quantities of the Kähler potential $K(\hat{\varphi}, \hat{\bar{F}}, \hat{\varphi}, \hat{F})$. Due to the essential role of the field F a part of the resulting dynamics can be unstable in the minimum of the effective potential. Thus we add additional terms to the action, all of them including explicit space-time derivatives. Obviously they do not change the effective potential, but they allow to add derivative terms of the order $\mathcal{O}(p^n)$ ($n \geq 2$) and to remove all instabilities. This procedure is possible as $F_0 \neq 0$.

In this way we are able to prove that any effective potential of the form (4.20) with $F_0 \neq 0$ can be provided with stable dynamics. Nevertheless, the concrete expressions will look rather undetermined in general. Thus we should point out that (4.20) does not fix all coupling constants. Indeed, different actions should be considered as equivalent if they have the following properties in common:

- The value of the effective potential in the minimum is the same.
- The vacuum expectation values of the fields φ and F are the same.
- In a systematic expansion around the ground-state, the dominant couplings (including the spectrum) are the same.

In a first step we derive the momentum-expansion of the action (4.4) with Kähler and superpotential as given in (4.14) and (4.15). This expansion stops at order $\mathcal{O}(p^4)$. The kinetic term for $\hat{\psi}$ is given by

$$\mathcal{L}^{(1)} = \frac{i}{2} (g_{\hat{\varphi}\hat{\varphi}} + g_{\hat{\varphi}\hat{\varphi},\hat{F}}\hat{F}_0 + g_{\hat{\varphi}\hat{\varphi},\hat{\bar{F}}}\hat{\bar{F}}_0) \hat{\psi} \sigma^\mu \overleftrightarrow{\partial}_\mu \hat{\psi} . \quad (4.26)$$

With (4.19) this simply translates into

$$\mathcal{L}^{(1)} = \frac{i}{2} (\tilde{g}_{\varphi\varphi} + \tilde{g}_{\varphi\varphi,F}F_0 + \tilde{g}_{\varphi\varphi,\bar{F}}\bar{F}_0) \psi \sigma^\mu \overleftrightarrow{\partial}_\mu \bar{\psi} . \quad (4.27)$$

The derivatives of $\mathcal{O}(p^2)$ acting on the spinors are rather simple as well:

$$\begin{aligned}\mathcal{L}_{fer}^{(2)} &= (g_{\hat{\varphi}\hat{F}} + \frac{1}{2}g_{\hat{\varphi}\hat{F},\hat{F}}\hat{F}_0)\hat{\psi}\square\hat{\psi} + (g_{\hat{\varphi}\hat{F}} + \frac{1}{2}g_{\hat{\varphi}\hat{F},\hat{F}}\hat{F}_0)\hat{\bar{\psi}}\square\hat{\bar{\psi}} \\ &= \frac{1}{2}(\tilde{g}_{\varphi F}\psi\square\psi + \tilde{g}_{\bar{\varphi}\bar{F}}\bar{\psi}\square\bar{\psi})\end{aligned}\tag{4.28}$$

In the second line we introduced the new notation

$$\tilde{g}_{\varphi F} = \frac{1}{9}\varphi_0^{-\frac{4}{3}}(2g_{\hat{\varphi}\hat{F}} + g_{\hat{\varphi}\hat{F},\hat{F}}\hat{F}_0)\tag{4.29}$$

This new component of the “metric” \tilde{g} as well as further components to be defined below are no longer derivatives of a Kähler potential, but optimized to economize writing. The bosonic part of $\mathcal{O}(p^2)$ turns out to be rather complicated. After transformation to the defining fields it can be written as

$$\begin{aligned}\mathcal{L}_{sc}^{(2)} &= \left(\tilde{g}_{\varphi\bar{\varphi}} + \frac{4}{9}\tilde{g}_{F\bar{F}}(\bar{\varphi}\varphi)_0^{-1}(\bar{F}F)_0 + (\tilde{g}_{\varphi\bar{\varphi},F}F_0 - \frac{2}{3}\tilde{g}_{\varphi\bar{F}}(\bar{\varphi}^{-1}\bar{F})_0 + \text{h. c.})\right)\partial_\mu\bar{\varphi}\partial^\mu\varphi \\ &\quad + \tilde{g}_{F\bar{F}}\partial_\mu\bar{F}\partial^\mu F + (\tilde{g}_{\varphi F}\partial_\mu F\partial^\mu\varphi + \text{h. c.}) \\ &\quad + \left((\tilde{g}_{\varphi\bar{F}} - \frac{2}{3}\tilde{g}_{F\bar{F}}(\varphi^{-1}F)_0)\partial_\mu\bar{F}\partial^\mu\varphi + \text{h. c.}\right) \\ &\quad + \left((\tilde{g}_{\varphi\varphi} - \frac{2}{3}\tilde{g}_{\varphi F}(\varphi^{-1}F)_0)\partial_\mu\varphi\partial^\mu\varphi + \text{h. c.}\right).\end{aligned}\tag{4.30}$$

The new quantities introduced here are:

$$\tilde{g}_{F\bar{F}} = \frac{1}{9}(\bar{\varphi}\varphi)_0^{-\frac{2}{3}}g_{\hat{F}\hat{F}}\tag{4.31}$$

$$\tilde{g}_{\varphi\bar{F}} = \frac{1}{9}\hat{F}_0(\bar{\varphi}\varphi)_0^{-\frac{2}{3}}g_{\hat{\varphi}\hat{F},\hat{F}}\tag{4.32}$$

$$\tilde{g}_{\varphi\varphi} = \frac{1}{9}\varphi_0^{-\frac{4}{3}}\hat{F}_0g_{\hat{\varphi}\hat{\varphi},\hat{F}}\tag{4.33}$$

Finally the terms $\mathcal{O}(p^3)$ and $\mathcal{O}(p^4)$ read:

$$\mathcal{L}^{(3)} = -\frac{i}{2}\tilde{g}_{F\bar{F}}\psi\sigma^\mu\overset{\leftrightarrow}{\partial}_\mu\square\bar{\psi}\tag{4.34}$$

$$\mathcal{L}^{(4)} = \tilde{g}_{F\bar{F}}\square\bar{\varphi}\square\varphi\tag{4.35}$$

Assume now that we have found the correct ground-state of $N = 1$ SYM from the effective potential (4.20), which corresponds to a certain choice of the coupling constants $\tilde{\alpha}_{mn}$ in (4.19). The resulting dynamics may have instabilities in momentum space in (4.30)

and (less important) in (4.35). In addition, (4.30) has many off-diagonal terms that we might want to cancel. In a first step we can use the freedom in the choice of the $\tilde{\alpha}_{mn}$ mentioned above. In fact, there is one term only that we cannot change in this way: The coefficient of $\mathcal{L}^{(1)}$ and the corresponding part of $\partial_\mu \bar{\varphi} \partial^\mu \varphi$ in $\mathcal{L}^{(2)}$ (cf. the comments at the end of this section). All other terms in the momentum-expansion depend on derivatives of the Kähler potential that do not appear in the effective potential. This follows from the fact that the effective potential depends on $\tilde{g}_{\varphi\bar{\varphi}}$ and derivatives thereof, only. Thus we can expect that most terms in the momentum expansion are not fixed by some choice of the ground-state. Nevertheless, this freedom may not be sufficient to obtain an acceptable dynamical behavior. Then we have to add terms involving explicit space-time derivatives according to the following rule [?]: To change the behavior of the Lagrangian at order $\mathcal{O}(p^n)$ we add a term with n space-time derivatives.

Inspecting the actions (4.4) and (4.6) together with the conditions (4.8) and (4.9) one finds:

- The superpotential remains a polynomial function in the fields.
- The dynamics to order $\mathcal{O}(p^2)$ cannot be changed by the action of the type (4.4), as a term $\propto \Psi_0 \square \bar{\Psi}_0$ does not exist.

Using the action (4.6), the new Lagrangian to add has the form

$$\mathcal{L}_{add} = \int d^4x \int d^4\theta \sum_{k=1}^{\infty} d_k (\Psi_0 \bar{\Psi}_0)^{-k} (\partial_\mu \Psi_0 \partial^\mu \square^{k-1} \bar{\Psi}_0) . \quad (4.36)$$

If d_1 can be chosen zero we can write instead of (4.36)

$$\mathcal{L}_{add} = - \int d^4x \int d^4\theta \sum_{k=2}^{\infty} d_k (\Psi_0 \bar{\Psi}_0)^{-k} (\square \Psi_0 \square^{k-1} \bar{\Psi}_0) , \quad (4.37)$$

which is now part of the action (4.4). This leads to the following new terms bilinear in the fields:

$$\begin{aligned} \mathcal{L}_{add}^{(2)} = & \int d^4x \frac{d_1}{9} (\bar{\varphi}\varphi)_0^{-1} ((\bar{F}F)_0 (\bar{\varphi}\varphi)_0^{-1} \partial_\mu \bar{\varphi} \partial^\mu \varphi + \partial_\mu \bar{F} \partial^\mu F \\ & - (F\varphi^{-1})_0 \partial_\mu \varphi \partial^\mu \bar{F} - (\bar{F}\bar{\varphi}^{-1})_0 \partial_\mu \bar{\varphi} \partial^\mu F) \end{aligned} \quad (4.38)$$

$$\mathcal{L}_{add}^{(2k+1)} = - \int d^4x \frac{id_k}{18} (\bar{\varphi}\varphi)_0^{-\frac{k+2}{3}} \psi \sigma^\mu \partial_\mu \square^k \bar{\psi} \quad (4.39)$$

$$\begin{aligned}\mathcal{L}_{add}^{(2k)} = & - \int d^4x (\bar{\varphi}\varphi)_0^{-\frac{k+1}{3}} \left(\left(\frac{(k+2)^2}{9} d_k (\bar{\varphi}\varphi)_0^{-\frac{4}{3}} (\bar{F}F)_0 - d_{k-1} \right) \varphi \square^k \bar{\varphi} + d_k (\bar{\varphi}\varphi)_0^{-\frac{1}{3}} F \square^k \bar{F} \right. \\ & \left. - \frac{k+2}{3} d_k (\bar{\varphi}\varphi)_0^{-\frac{1}{3}} \left((F\varphi^{-1})_0 \varphi \square^k \bar{F} + (\bar{F}\bar{\varphi}^{-1})_0 \bar{\varphi} \square^k F \right) \right)\end{aligned}\tag{4.40}$$

In equation (4.39) the index runs from $1 \cdots \infty$, in equation (4.40) from $2 \cdots \infty$. The proof that we can obtain stable dynamics is now rather trivial:

- If the dynamics to order $\mathcal{O}(p^2)$ are not stable we choose $d_1 > 0$ in such a way that the eigenvalues of the dynamics are all positive. This is always possible as
 1. (4.38) has no unstable terms. This follows from $F_0 > 0$ and $\varphi_0 < 0$. On the other hand it adds stable terms to all possible combinations.
 2. d_1 is not constrained.
- If $\tilde{g}_{F\bar{F}} > 0$ the dynamics to order $\mathcal{O}(p^4)$ in φ have the wrong sign. Also, the term in d_1 adds unstable terms in all combinations at this order. These terms are suppressed already, if $p^2 \ll \Lambda^2$. Nevertheless we might want to change this. We thus choose:

$$d_2 > \frac{9}{16} (\bar{\varphi}\varphi)_0^{\frac{4}{3}} (\bar{F}F)_0^{-1} (9(\bar{\varphi}\varphi)_0 \tilde{g}_{F\bar{F}} + d_1) \tag{4.41}$$

If $\tilde{g}_{F\bar{F}} < 0$ we relax the constraint for d_2 to equation (4.42) below.

- Now the dynamics are stable to $\mathcal{O}(p^4)$, but d_2 adds unstable terms to order $\mathcal{O}(p^6)$. In general all unstable terms are getting removed by the recursive formula

$$d_k > \frac{9}{(k+2)^2} (\bar{\varphi}\varphi)_0^{\frac{4}{3}} (\bar{F}F)_0^{-1} d_{k-1} . \tag{4.42}$$

In addition to the suppression by orders of Λ the new coupling constants can thus be chosen suppressed by a factor k^{-2} .

We point out again that this procedure is consistent with the ansatz (4.4) if d_1 can be chosen zero. If so, the dynamics to $\mathcal{O}(p^2)$ must be stable without further modifications.

To cancel unexpected off-diagonal contributions to the kinetic Lagrangian, more complicated terms including explicit space-time derivatives can be introduced. A possible set, which allows to cancel any off-diagonal contribution, has been given in [?].

There exists one term in the dynamics that remains unchanged by any choice of the d_k : $\mathcal{L}^{(1)}$. Therefore it is a strict condition on the effective potential that the coefficient of this term does not vanish in the minimum. But this simply means

$$\tilde{g}_{\varphi\bar{\varphi}} + \tilde{g}_{\varphi\bar{\varphi},F} F_0 + \tilde{g}_{\varphi\bar{\varphi},\bar{F}} \bar{F}_0 = -\tilde{g}_{\varphi\bar{\varphi}} + c \left(\bar{F}_0^{-1} \log \left(\frac{z\varphi}{\Lambda^3} \right) + \text{h. c.} \right) = (V_{\text{eff}})_0 \neq 0 , \tag{4.43}$$

where we have used (4.21). This condition just says that the effective potential in the minimum must be non-vanishing: $(V_{\text{eff}})_0 \neq 0$. Nevertheless, the sign is not fixed by stability arguments. The constraint then says that our models are actually divided in two classes, one having $(V_{\text{eff}})_0 > 0$, the other one $(V_{\text{eff}})_0 < 0$. Starting from a model of one class it is impossible to deform the latter into a model of the other class by a continuous transformation.

5 The Role of Dynamical Symmetry Breaking

We have shown in the last section how to construct a non-local effective action for SYM. We have proven that the inclusion of higher order derivatives breaks supersymmetry dynamically and we finally showed that any consistent minimum of the effective potential can be equipped with stable dynamics. Nevertheless several questions remain and in this section we want to discuss some of them, especially:

1. Different semi-classical and/or perturbative arguments suggest that supersymmetry should be unbroken in SYM theories. Why don't these arguments exclude our solution?
2. Closely related to the first question is the behavior of the potential for $\varphi \rightarrow 0$ and $F \rightarrow 0$. The first limit is interesting in the action by Veneziano and Yankielowicz as well: There the bosonic potential has a second minimum for $\varphi = 0$, which led to speculations about a chirally symmetric minimum [?]. Do similar problems appear in our calculation?
3. We have defined our theory as an effective action obtained from a source extension. How does the effective action depend on these sources? Especially, we can study the limit of constant sources, which is formally identical to soft supersymmetry breaking. What happens to the Goldstino in this case?

5.1 Singularities of V_{eff} and DSB

We address the second point first. The bosonic potential is non-singular for $\varphi = 0$ if $m \leq -\frac{1}{2}$ in (4.25). This allows a term $(\bar{F}F)^{\frac{1}{2}}$ to stabilize the potential for large F and in principle a consistent minimum should be possible. Nevertheless, the meaning of the limit $\varphi \rightarrow 0$ remains unclear: As in the case of Veneziano and Yankielowicz, the dynamics as well as the fermionic potential diverge at this point. Also, the potential is now restricted to a simple form that we do not expect for an effective theory. Thus this restriction should not play any role in the discussion.

More interesting is the behavior for $F \rightarrow 0$, as this limit can be important in the interpretation of dynamical supersymmetry breaking. The bosonic potential is regular in this limit for $m \geq -1$. Again, this should be sufficient to construct a consistent minimum in the sense of the conditions (4.21) and (4.23). Similar to the situation of $\varphi \rightarrow 0$, $m \geq -1$ is not

enough to ensure a regular limit of the whole potential. Also, the dynamics will diverge in the limit, which however is true for any model of the type discussed in this work. Regularity of the whole potential would require $m \geq 0$, a constraint that cannot be fulfilled.

The typical behavior expected is thus the divergence of the potential in both limits, $\varphi \rightarrow 0$ and $F \rightarrow 0$. The divergence for $\varphi \rightarrow 0$ is related to chiral symmetry breaking and thus to the impossibility of massive spectra in the fermionic sector if $\varphi = 0$. Similarly we can interpret the divergence for $F \rightarrow 0$ with the massive spectrum in boson sector. Although the relation between the vacuum expectation value of $\text{Tr } F_{\mu\nu} F^{\mu\nu}$ and the formation of the mass-gap in this sector is less well understood than the analogue relation in the fermionic sector, it seems that the infrared divergences cannot be removed if this vacuum expectation value vanishes [?, ?]. For further discussions of the interpretation of $F_0 \neq 0$ within supersymmetry we refer to [?].

Furthermore the divergence of the potential for $F \rightarrow 0$ has important implications in the interpretation of dynamical supersymmetry breaking (the first question formulated above). Indeed, we should be able to explain why semi-classical analysis (instantons, monopoles etc.) as well as topological arguments (Witten index) do not lead to the correct result. It has been discussed in [?] (section 6.1) why these arguments cannot exclude dynamical symmetry breaking. In summary there exist the following two loopholes:

1. The supersymmetric state postulated by the Witten index indeed does exist, but this state is not the true ground-state. This is consistent with all symmetries: Supersymmetry tells us, that the vacuum energy must be positive semi-definite. This does not mean that the effective potential is positive semi-definite (cf. section 3, sect. 6.1 of [?] and sect. 2.2.1 of [?]).

In this scenario the semi-classical analysis is an expansion around the wrong state and fails to capture all relevant effects. Any of our non-local models with a regular potential for $F \rightarrow 0$ appears as a realization of this situation. Trivially it exists if $(V_{\text{eff}})_0 < 0$, only.

2. The supersymmetric ground-state does not exist at all. In this context it is important to notice that all arguments in favor of unbroken supersymmetry are brought forward in an infrared regularized region, either in the finite volume (instantons and similar, Witten index) or with a momentum cutoff (Wilsonian effective action). As these approaches cannot solve the fundamental infrared problem of non-Abelian gauge theories, but just remove it by brute force, we indeed expect dynamical effects not captured therein. We interpret these effects with the formation of the massive spectrum in the boson sector and the dynamics of the glue-ball. Our result suggests that these dynamical effects are not consistent with unbroken supersymmetry. But as any consistent quantum-field theory must solve the infrared problem by appropriate dynamical effects, the perturbatively supersymmetric ground-state is removed by a singularity and replaced by the correct one enforcing dynamical symmetry breaking.

According to the discussion of the limit $F \rightarrow 0$ we expect a behavior as outlined in point 2. As the supersymmetric state is getting removed by a singularity, a restriction on the value of the effective potential in the minimum does not exist.

It has already been pointed out in [?] that the kinetic term of the Goldstino has the wrong sign if $(V_{\text{eff}})_0 < 0$. In principle this is not an instability of the dynamics, but it leads to difficulties in the interpretation of the action if one would like to couple additional fields. Together with the singularity in the potential for $F \rightarrow 0$ we thus expect $(V_{\text{eff}})_0 > 0$.

The same coefficient as in $\mathcal{L}^{(1)}$ appears in $\mathcal{L}^{(2)}$ as well. There exists no strict statement that the dynamics to order $\mathcal{O}(p^2)$ are unstable if $(V_{\text{eff}})_0 < 0$. Nevertheless, $(V_{\text{eff}})_0 > 0$ leads to additional stable contributions in $\mathcal{L}^{(2)}$ and we can expect stable dynamics even when choosing $d_1 = 0$ in (4.36). This would lead to the gratifying result that the $N = 1$ SYM effective action can be written in the simple form (4.4).

5.2 Local Coupling and Hysteresis Effects: Towards an Understanding of DSB?

Finally, we want to say a few words about the dependence of the effective action on its sources and give an outlook on interesting topics to be considered.

Of course, all sources have been removed by Legendre transformation. Nevertheless, we can study the limit of (almost) constant sources, which corresponds to softly broken supersymmetry. In this limit we formally obtain a parametrical dependence of the effective action on the constant part of the sources. This parametrical dependence is not constrained by holomorphic dependence and the different components of the sources do not form a chiral superfield (ref. [?], sect. 2 for details). This can be seen as a consequence of renormalization, which does not respect holomorphic dependence in the local coupling superfield as the β -function is essentially non-holomorphic. Notice again that we are talking about a fully renormalized theory, where the Wilsonian 1-loop β -function is obsolete.

Especially interesting is the limit of a constant gluino mass m in (4.1). Actually the true ground-state of the theory can be determined by considering hysteresis lines of this type only (ref. [?] and references therein). Thus we should consider all quantities of our effective action to depend parametrically on a mass parameter m . Of course, the same applies to the Yang-Mills coupling constant g . This extension alone does not necessarily lead to additional constraints on the effective action. However, by performing the limit $m \rightarrow \text{const.}$ we break supersymmetry softly and thus the (pseudo-)Goldstino should become massive. This is possible by including a spurion field, only. In contrast to the spurion field of the fundamental theory, $J = \theta^2 m$, the spurion field of the effective theory cannot be chiral. This is again a consequence of the consistent interpretation of the superpotential: In the context of our effective action, it is impossible to obtain stable couplings from the superpotential. There exists no procedure (as e.g. the elimination of the auxiliary fields) that would transform a holomorphic expression like $m\varphi F$ into a non-holomorphic but stable

one ($|m|^2|\varphi|^2$). Obviously, this is not restricted to the fundamental superpotential but also applies to contributions from spurion fields. Thus, the simplest possibility would be to add a term of the form

$$\mathcal{L}_{spurion} = \int d^4x \int d^4\theta |M(m)|^2 \Psi_0 \bar{\Psi}_0 . \quad (5.1)$$

Equation (5.1) consistently modifies the mass of φ as well as the minimum condition (4.23). On the other hand the second variation with respect to ψ remains unchanged and thus the (pseudo-)Goldstino receives a mass. To our present knowledge the fact that the effective spurion field must be non-holomorphic cannot be obtained in the Wilsonian action.

Of course the realization of all symmetries should be studied carefully for this extension. This essentially includes effects from local coupling constants and non-perturbative effects beyond the instanton-approximation (i.e. beyond the NSVZ β -function). It is beyond the scope of this paper to answer all these questions, but it should be noted that interesting new constraints on the general form of the effective action can be obtained by this means: Recent results from perturbation theory showed that local couplings in SYM theories lead to a new anomaly [?, ?, ?]. This anomaly is closely related to the breaking of holomorphic dependence by higher order quantum effects. On the other hand we expect that exactly these effects on the non-perturbative level are responsible for dynamical supersymmetry breaking: Most calculations in non-perturbative supersymmetry are performed in the infrared regularized region, where holomorphic dependence is restored by the Wilsonian 1-loop β -function. The non-holomorphic terms are obtained by a formal switch from the Wilson action to the (infrared regularized) quantum effective action. But neither the Wilson nor the non-holomorphic (NSVZ) β -function is infrared stable (notice that they are both perturbative β -functions). Thus an acceptable infrared behavior essentially includes corrections to the β -function that could be related to an extension of the above mentioned symmetry properties to the non-perturbative region.

6 Conclusions

In this paper we have introduced the extension of the Veneziano-Yankielowicz effective action to a non-local action. Our modification affects the non-holomorphic parts (Kähler potential) of the action only: While the Kähler metric from the Veneziano-Yankielowicz effective action is solely a function of the gluino-condensate, our Kähler potential includes terms in the gluon-condensate (the “auxiliary” field F of the effective superfield) as well. As a consequence the effective potential depends on infinitely many coupling constants related to arbitrary powers in φ and F . As the superpotential is unchanged compared to the Veneziano-Yankielowicz action, our extension is complementary to recent investigations on the holomorphic terms that build on the work by Dijkgraaf and Vafa [?, ?, ?, ?].

The main characteristics of our effective action are:

1. The “auxiliary” field F is promoted to an independent physical field. To obtain the spectrum of the theory it is not getting eliminated. Consequently, the spectrum of our model contains two complex scalars φ and F and one spinor ψ . F is interpreted with the glue-ball and the new dynamical effects represent the consequence of the formation of a massive spectrum in the boson sector.
2. To arrive at a consistent formulation of point 1, the “upside-down” potential for F in the Veneziano-Yankielowicz Lagrangian $V_F \propto -F\bar{F}$ has to be replaced. The correct potential must have a minimum in F instead of a maximum. Technically this is done by formulating the effective potential in terms of the two fields $\Psi_0 = (\Phi)^{\frac{1}{3}}$ and $\Psi_1 = \bar{D}^2\bar{\Psi}_0$ (Φ denotes the effective glue-ball superfield).
3. The spectrum of the theory does not follow from the superpotential, but from the generalized Kähler potential. The superpotential as a holomorphic function in the fields never defines stable couplings in the complex fields. If the highest components of the chiral fields are auxiliary, stable couplings are obtained after elimination of these fields. As our action does not allow this elimination, the Kähler potential must adopt the role of the superpotential. This is possible due to its rich structure. The superpotential reduces to the generator of the anomalies (WZ-term) and has no dominant influence on the spectrum.
4. Supersymmetry breaks dynamically. There are three different ways to establish this result:
 - (a) A strict proof of the statement follows from the symmetries of the effective potential: The first variation of the effective potential with respect to φ is equivalent to the mass of ψ . Thus the mass of ψ is identically zero in the minimum and must be interpreted as a Goldstino mode.
 - (b) The Veneziano-Yankielowicz Lagrangian has a supersymmetric maximum in the “auxiliary”-field potential (the supersymmetric vacua in the interpretation of ref. [?]). As the Veneziano-Yankielowicz Lagrangian is complete up to order $\bar{F}F$ in the “auxiliary”-field potential, the maximum cannot be turned into a minimum. Thus the minimum must be at a different value of F , $F_0 \neq 0$.
 - (c) $F_0 \neq 0$ plays an important role in the dynamics. Due to this vacuum expectation value, new terms including explicit space-time derivatives follow. These terms can be important to stabilize the dynamics around the correct ground-state.

The breaking-mechanism is of essentially non-perturbative nature and is not excluded by different contrarian results (Instanton calculations, Witten index etc.).

5. Any ground-state of the effective potential can be endowed with stable dynamics. This includes terms of arbitrary order in the momentum (non-local effective action).

Though we are convinced that our effective action can answer several open questions about the non-perturbative dynamics of SYM theories different problems remain. First we have addressed in this work the invariant terms of the action, only. Especially when considering non-vanishing values of the source-extension this is not sufficient. Thus we should include the complete (anomalous) symmetry structure of the effective action for local couplings as well as for soft supersymmetry breaking. Recent results from perturbation theory [?, ?, ?] suggest that this extension is closely related to the breaking of holomorphic dependence by quantum corrections. As the impact of non-holomorphic contributions to the potential is certainly a main result of our calculations, a better understanding of these terms could be of main importance. Finally, we have shown that the semi-classical analysis of SYM theories as instanton calculations deal with an expansion around an unstable state. Classical field configurations responsible for the correct ground-state are unknown, but these field configurations should be related to the glue-ball dynamics. It would be very interesting to investigate this questions, which could be helpful to understand the non-perturbative dynamics of non-supersymmetric gauge-theories as well.

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